STAR-Net: An Interpretable Tensor Representation Network for Hyperspectral Image Denoising

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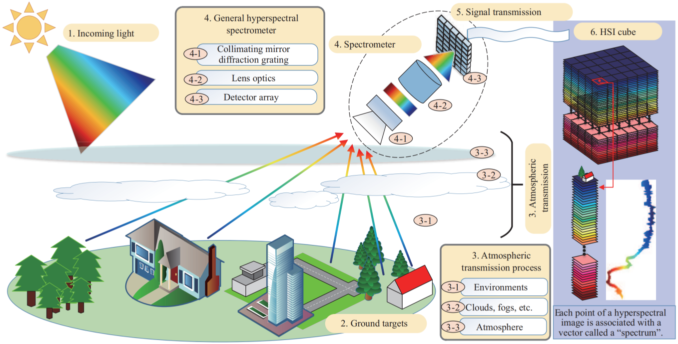
Outline

HSI

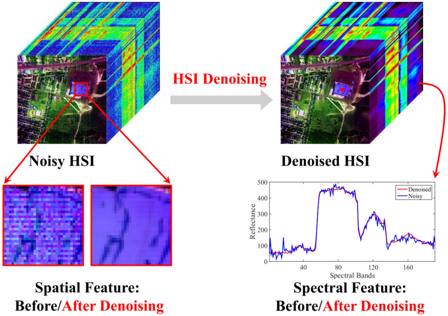
- Hyperspectral image (HSI): Rich spectral and spatial information



- Board applications: Agriculture, astronomy, geosciences, medicine
- Various tasks: Denoising, classification, detection, fusion, unmixing
- `https://github.com/xianchaoxiu/Hyperspectral-Imaging`

Denoising

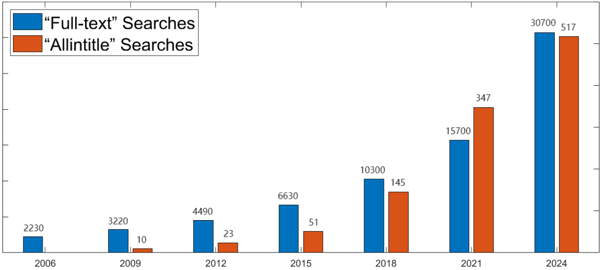
- HSI denoising/HSI restoration



- Taxonomy of existing methods
  - Model-driven: Non-local, TV, sparse coding, low-rank representation
  - Data-driven: CNN, hybrid networks, unsupervised networks
  - Model-data-driven

Tensor

- The development of "tensor hyperspectral" on Google Scholar

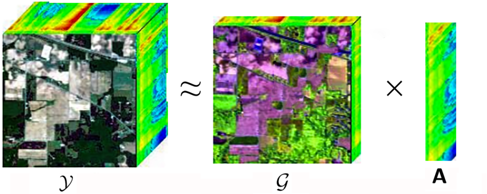


- What are the advantages of tensor modeling?
  - Excellent data representation
  - Various tensor decomposition
  - (possibly) Low computational complexity

Motivation

- General subspace tensor representation framework

$$
\begin{aligned}
\text{(P)} \quad & \min_{\mathcal{G},\mathbf{A}} \ \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda\Omega(\mathcal{G}) \\
& \text{s.t.} \ \ \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}
$$



- Recent surveys
  - Liu-Li-Wang-Tao-Du-Chanussot, SCIS, 2023
  - Wang-Hong-Han-Li-Yao-Gao, IEEE GRSM, 2023

Motivation

- Xiong-Zhou-Tao-Lu-Zhou-Qian, IEEE TIP, 2022

$$
\begin{aligned}
(\mathrm{P}) \quad \min_{\mathcal{G},\mathbf{A}} \quad & \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda\Omega(\mathcal{G}) \\
\text{s.t.} \quad & \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\min_{\mathcal{G},\mathcal{B}_i,\mathbf{A}} \quad & \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda \sum_i (\phi(\mathcal{G}, \mathcal{B}_i) + \gamma_1 \|\mathcal{B}_i\|_1) \\
\text{s.t.} \quad & \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}
$$

- Multidimentional representation

$$\phi(\mathcal{G}, \mathcal{B}_i) = \frac{1}{2}\|\mathcal{R}_i\mathcal{G} - \mathcal{B}_i \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3\|_{\mathrm{F}}^2$$

How to characterize priors? How to develop algorithms?

Outline

Model

- Sparse tensor aided representation (STAR)

$$
\begin{aligned}
\text{(P)} \quad & \min_{\mathcal{G},\mathbf{A}} \; \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda\Omega(\mathcal{G}) \\
& \text{s.t.} \;\; \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\text{(STAR)} \quad & \min_{\mathcal{G},\mathcal{B}_i,\mathbf{A}} \; \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda \sum_i \left(\phi(\mathcal{G}, \mathcal{B}_i) + \gamma_1\|\mathcal{B}_i\|_1 + \gamma_2\|\mathcal{B}_i\|_*\right) \\
& \text{s.t.} \quad \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\text{(STAR-S)} \quad & \min_{\mathcal{G},\mathcal{S},\mathcal{B}_i,\mathbf{A}} \; \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A} - \mathcal{S}\|_{\mathrm{F}}^2 + \mu\|\mathcal{S}\|_1 \\
& \qquad + \lambda \sum_i \left(\phi(\mathcal{G}, \mathcal{B}_i) + \gamma_1\|\mathcal{B}_i\|_1 + \gamma_2\|\mathcal{B}_i\|_*\right) \\
& \text{s.t.} \quad \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}
$$

Algorithm

- Alternating direction method of multipliers (ADMM)

$$\min_{\mathcal{G},\mathcal{B}_i,\mathcal{L}_i,\mathbf{A}} \quad \frac{1}{2}\|\mathcal{Y}-\mathcal{G}\times_3\mathbf{A}\|_{\mathrm{F}}^2+\lambda\sum_i\left(\phi(\mathcal{G},\mathcal{B}_i)+\gamma_1\|\mathcal{B}_i\|_1+\gamma_2\|\mathcal{L}_i\|_*\right)$$

$$\text{s.t.} \quad \mathbf{A}^\top\mathbf{A}=\mathbf{I},\ \mathcal{L}_i=\mathcal{B}_i$$

$$\Downarrow$$

$$\begin{aligned} L_\beta(\mathcal{G},\mathcal{B}_i,\mathcal{L}_i,\mathbf{A},\mathcal{P}_i) &= \frac{1}{2}\|\mathcal{Y}-\mathcal{G}\times_3\mathbf{A}\|_{\mathrm{F}}^2+\langle\mathcal{P}_i,\mathcal{L}_i-\mathcal{B}_i\rangle \\ &\quad +\lambda\sum_i\left(\phi(\mathcal{G},\mathcal{B}_i)+\gamma_1\|\mathcal{B}_i\|_1+\gamma_2\|\mathcal{L}_i\|_*\right)+\frac{\beta}{2}\|\mathcal{L}_i-\mathcal{B}_i\|_{\mathrm{F}}^2 \end{aligned}$$

- From iterative optimization to deep unrolling
  - Gregor-LeCun, ICML, 2010
  - Yang-Sun-Li-Xu, NIPS, 2016
  - Monga-Li-Eldar, IEEE SPM, 2021
  - Elad-Kawar-Vaksman, SIIMS, 2023

STAR-Net

- Update $\mathcal{G}$-block

$$
\begin{aligned}
\mathcal{G}^{k+1} = \arg\min_{\mathcal{G}} \ & \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}^k\|_{\mathrm{F}}^2 \\
& + \frac{\lambda}{2}\sum_i \|\mathcal{R}_i\mathcal{G} - \mathcal{B}_i^k \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3\|_{\mathrm{F}}^2
\end{aligned}
$$

$$\Downarrow$$

$$\mathcal{G}^{k+1} = (\mathcal{I} + \lambda \sum_i \mathcal{R}_i^\top \mathcal{R}_i)^{-1}(\lambda \sum_i \mathcal{R}_i^\top \mathcal{B}_i^k \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3 + \mathcal{Y} \times_3 \mathbf{A}^{k\top})$$

$$\Downarrow$$

$$\mathcal{G}^{k+1} = \mathcal{E}_1 * \mathcal{E}_2$$

$$\Downarrow$$

$$\mathcal{G}^{k+1} = \mathrm{LargNet}(\mathcal{E}_1, \mathcal{E}_2)$$

STAR-Net

- Update $\mathcal{B}_i$-block

$$\mathcal{B}_i^{k+1} = \arg\min_{\mathcal{B}_i} \frac{\lambda}{2}\|\mathcal{R}_i\mathcal{G}^{k+1} - \mathcal{B}_i \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3\|_{\mathrm{F}}^2$$
$$+ \frac{\beta}{2}\|\mathcal{L}_i^k - \mathcal{B}_i + \mathcal{P}_i^k/\beta\|_{\mathrm{F}}^2 + \lambda\gamma_1\|\mathcal{B}_i\|_1$$
$$\Downarrow$$
$$\mathcal{B}_i^{k+1} = \arg\min_{\mathcal{B}_i} \frac{1}{2}\|(\beta\mathcal{I} + \lambda\mathcal{I} \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3)\mathcal{B}_i$$
$$- (\lambda\mathcal{R}_i\mathcal{G}^{k+1} + \beta\mathcal{L}_i^k + \mathcal{P}_i^k)\|_{\mathrm{F}}^2 + \lambda\gamma_1\|\mathcal{B}_i\|_1$$
$$\Downarrow$$
$$\mathcal{B}_i^{k+1} = \mathrm{Shrink}(\mathcal{F}_i, \lambda\gamma_1/\nu)$$
$$\Downarrow$$
$$\mathcal{B}_i^{k+1} = \mathrm{sgn}(\mathcal{F}_i)\mathrm{ReLU}(|\mathcal{F}_i| - \lambda\gamma_1/\nu)$$

STAR-Net

- Update $\mathbf{A}$-block

$$\mathbf{A}^{k+1} = \arg\min_{\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \frac{1}{2}\|\mathcal{Y} - \mathcal{G}^{k+1} \times_3 \mathbf{A}\|_{\mathrm{F}}^2$$
$$\Downarrow$$
$$\mathbf{A}^{k+1} = \mathbf{U}\mathbf{V}^\top$$
$$\Downarrow$$
$$\mathbf{A}^{k+1} = \mathrm{LargNet}(\mathbf{U}, \mathbf{V}^\top)$$

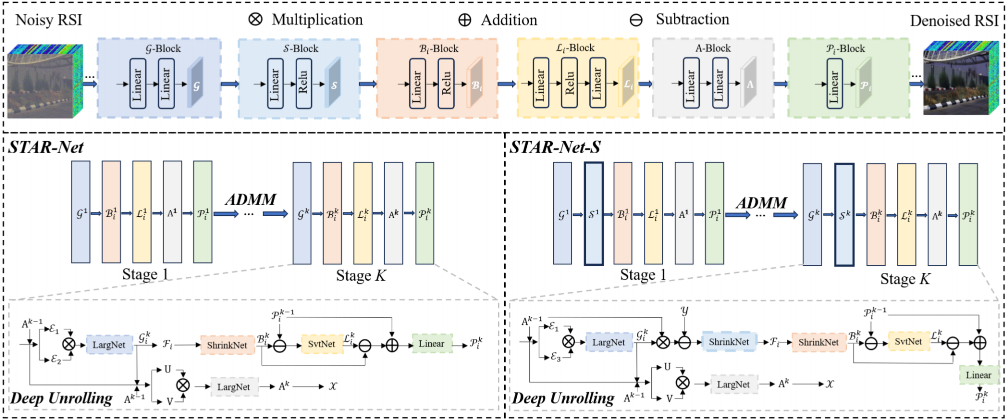
- Update $\mathcal{P}_i$-block

$$\mathcal{P}_i^{k+1} = \mathcal{P}_i^k + \beta(\mathcal{L}_i^{k+1} - \mathcal{B}_i^{k+1})$$
$$\Downarrow$$
$$\mathcal{P}_i^{k+1} = \mathrm{Linear}(\Theta_i)$$

STAR-Net

- Input: Noisy HSI $\mathcal{Y}$, parameters $\lambda, \beta, \gamma_1, \gamma_2, \nu$
- Initialize: $(\mathcal{G}^0, \mathcal{B}_i^0, \mathcal{L}_i^0, \mathbf{A}^0, \mathcal{P}_i^0)$
- While $k = 1, \ldots, K$ do
  - Update $\mathcal{G}$-block by
    $$\mathcal{G}^{k+1} = \mathrm{LargNet}(\mathcal{E}_1, \mathcal{E}_2)$$
  - Update $\mathcal{B}_i$-block by
    $$\mathcal{B}_i^{k+1} = \mathrm{ShrinkNet}(\mathcal{F}_i, \lambda\gamma_1/\nu)$$
  - Update $\mathcal{L}_i$-block by
    $$\mathcal{L}_i^{k+1} = \mathrm{SvtNet}(\mathcal{B}_i^{k+1} - \mathcal{P}_i^k/\beta, \lambda\gamma_2/\beta)$$
  - Update $\mathbf{A}$-block by
    $$\mathbf{A}^{k+1} = \mathrm{LargNet}(\mathbf{U}, \mathbf{V}^\top)$$
  - Update $\mathcal{P}_i$-block by
    $$\mathcal{P}_i^{k+1} = \mathrm{Linear}(\Theta_i)$$
- Output: Denoised HSI $\mathcal{X} = \mathcal{G}^{k+1} \times_3 \mathbf{A}^{k+1}$

Architecture



Outline

Setup

- Compared methods
  - BM3D: Dabov-Katkovnik-Egiazarian, IEEE TIP, 2007
  - BM4D: Maggioni-Katkovnik-Egiazarian-Foi, IEEE TIP, 2012
  - LLRT: Chang-Yan-Zhong, CVPR, 2017
  - LRTDTV: Wang-Chen-Han-He, RS, 2017
  - NGMeet: He-Yao-Li-Yokoya-Zhao-Zhang-Zhang, IEEE TPAMI, 2022
  - HSI-SDeCNN: Maffei-Haut-Paoletti-Plaza, IEEE TGRS, 2020
  - SMDS-Net: Xiong-Zhou-Tao-Lu-Zhou-Qian, IEEE TIP, 2022
  - RCILD: Peng-Wang-Cao-Zhao-Yao-Zhang-Meng, IEEE TGRS, 2024
- Implementation details
  - Training loss: $L = \|\mathrm{STAR\text{-}Net}(\mathcal{Y}) - \mathcal{X}\|_{\mathrm{F}}^2$
  - Training dataset: 100 HSIs from ICVL, data augmentation
  - Testing dataset: ICVL, Washington DC Mall
  - Hyperparameters: Learning rate=0.005, batch=2, epoch=300, K=6, dictionary=[9, 9, 9]
  - Training parameters: $\lambda, \beta, \mu, \gamma_1, \gamma_2, \mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3$

Synthetic

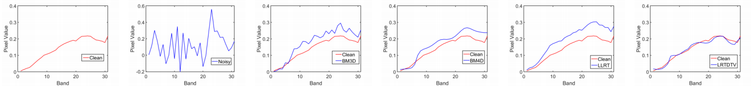
- Quantitative comparisons on ICVL

σ	Index	Noisy	Model-based methods					Deep learning-based methods				
			BM3D	BM4D	LLRT	LRTDTV	NGMeet	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
10	PSNR ↑	29.018	37.310	42.987	39.810	43.882	42.383	41.519	46.371	42.458	**47.286**	**47.345**
	SSIM ↑	0.521	0.924	0.973	0.962	0.979	0.966	0.969	0.985	0.987	**0.988**	**0.989**
	SAM ↓	0.229	0.121	0.080	0.045	0.077	0.074	0.075	**0.028**	0.044	**0.025**	**0.025**
30	PSNR ↑	21.591	32.582	37.630	34.250	38.245	36.791	36.840	42.337	38.514	**42.435**	**42.500**
	SSIM ↑	0.146	0.846	0.930	0.921	0.877	0.915	0.926	**0.972**	**0.971**	**0.972**	**0.972**
	SAM ↓	0.535	0.208	0.142	0.084	0.149	0.139	0.124	0.040	0.067	**0.039**	**0.038**
50	PSNR ↑	18.402	29.982	35.242	32.067	33.618	34.399	34.342	37.481	35.838	**39.853**	**39.963**
	SSIM ↑	0.042	0.790	0.888	0.899	0.862	0.887	0.893	0.907	**0.951**	**0.956**	**0.956**
	SAM ↓	0.779	0.264	0.190	0.107	0.195	0.177	0.134	0.066	0.092	**0.050**	**0.047**
70	PSNR ↑	18.126	28.654	33.586	30.746	30.565	32.389	32.794	37.197	33.980	**37.342**	**38.237**
	SSIM ↑	0.038	0.742	0.844	0.852	0.762	0.858	0.855	0.923	**0.930**	**0.943**	**0.943**
	SAM ↓	0.897	0.310	0.231	0.214	0.304	0.217	0.186	0.066	0.132	**0.058**	**0.055**
Ave.	PSNR ↑	21.784	32.132	37.361	34.218	36.578	36.490	36.374	40.463	37.212	**41.729**	**42.011**
	SSIM ↑	0.187	0.826	0.909	0.909	0.870	0.907	0.911	0.943	**0.953**	**0.965**	**0.965**
	SAM ↓	0.610	0.226	0.161	0.113	0.181	0.152	0.130	0.050	0.077	**0.043**	**0.041**

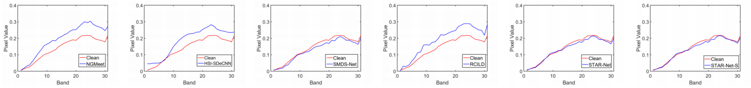
Synthetic

(a) Clean/PSNR (b) Noisy/18.792 (c) BM3D/30.578 (d) BM4D/33.410 (e) LLRT/30.746 (f) LRTDTV/32.993

(g) NGMeet/31.510 (h) HSI-SDeCNN/32.141 (i) SMDS-Net/36.641 (j) RCILD/33.546 (k) STAR-Net/38.248 (l) STAR-Net-S/38.293



(a) Clean (b) Noisy (c) BM3D (d) BM4D (e) LLRT (f) LRTDTV



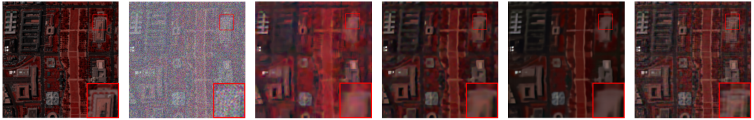
(g) NGMeet (h) HSI-SDeCNN (i) SMDS-Net (j) RCILD (k) STAR-Net (l) STAR-Net-S

Synthetic

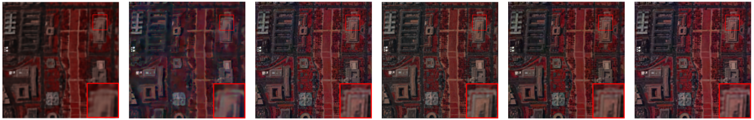
- Quantitative comparisons on WDC

σ	Index	Noisy	Model-based methods					Deep learning-based methods				
			BM3D	BM4D	LLRT	LRTDTV	NGMeet	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
10	PSNR ↑	28.561	36.142	38.435	39.540	40.887	42.712	40.756	43.616	28.968	**46.173**	**46.410**
	SSIM ↑	0.522	0.923	0.946	0.966	0.909	0.978	0.937	0.961	0.972	**0.980**	**0.982**
	SAM ↓	0.738	0.338	0.271	0.265	0.236	0.200	0.304	0.066	0.215	**0.043**	**0.042**
30	PSNR ↑	20.897	30.869	32.793	32.681	38.887	37.391	36.710	39.498	22.509	**39.924**	**39.950**
	SSIM ↑	0.150	0.784	0.834	0.858	0.888	0.854	0.832	0.916	0.882	**0.923**	**0.928**
	SAM ↓	1.020	0.519	0.445	0.387	0.335	0.260	0.399	**0.085**	0.243	**0.083**	**0.083**
50	PSNR ↑	17.778	28.882	30.901	30.470	35.250	36.160	35.062	36.014	22.160	**36.731**	**37.280**
	SSIM ↑	0.066	0.702	0.770	0.789	0.813	0.793	0.771	0.834	**0.872**	0.852	**0.871**
	SAM ↓	1.164	0.603	0.528	0.433	0.504	0.340	0.446	0.124	0.263	**0.122**	**0.121**
70	PSNR ↑	16.966	27.694	30.140	29.159	34.198	35.110	32.891	35.315	21.262	**35.964**	**36.286**
	SSIM ↑	0.051	0.657	0.740	0.761	0.776	0.761	0.602	0.811	**0.857**	0.831	**0.843**
	SAM ↓	1.205	0.652	0.560	0.433	0.547	0.500	0.980	0.135	0.290	**0.130**	**0.129**
Ave.	PSNR ↑	21.051	30.897	33.067	32.962	37.305	37.843	36.355	38.611	23.725	**39.698**	**39.982**
	SSIM ↑	0.197	0.766	0.823	0.844	0.847	0.846	0.786	0.881	0.896	**0.897**	**0.906**
	SAM ↓	1.032	0.528	0.451	0.380	0.406	0.325	0.532	0.102	0.253	**0.095**	**0.094**

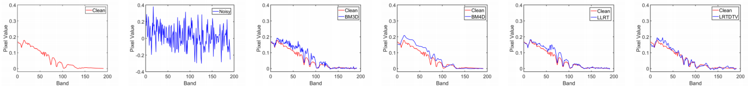
Synthetic



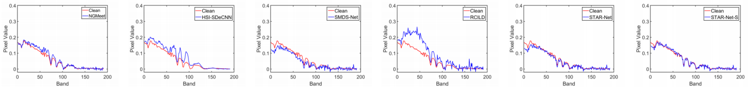
(a) Clean/PSNR (b) Noisy/17.778 (c) BM3D/28.882 (d) BM4D/30.901 (e) LLRT/30.470 (f) LRTDTV/35.250



(g) NGMeet/36.160 (h) HSI-SDeCNN/35.062 (i) SMDS-Net/36.014 (j) RCILD/22.160 (k) STAR-Net/36.731 (l) STAR-Net-S/37.280



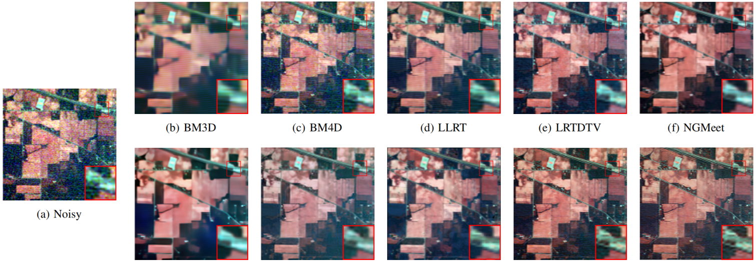
(a) Clean (b) Noisy (c) BM3D (d) BM4D (e) LLRT (f) LRTDTV



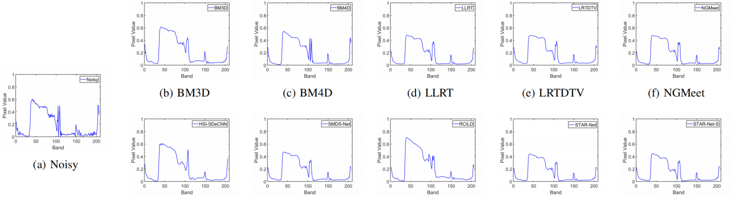
(g) NGMeet (h) HSI-SDeCNN (i) SMDS-Net (j) RCILD (k) STAR-Net (l) STAR-Net-S

Real-world

- Case study on Indian Pines



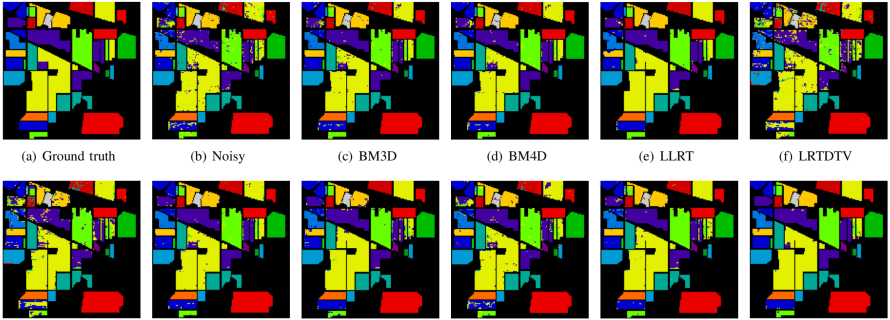
(a) Noisy (b) BM3D (c) BM4D (d) LLRT (e) LRTDTV (f) NGMeet (g) HSI-SDeCNN (h) SMDS-Net (i) RCILD (j) STAR-Net (k) STAR-Net-S



(a) Noisy (b) BM3D (c) BM4D (d) LLRT (e) LRTDTV (f) NGMeet (g) HSI-SDeCNN (h) SMDS-Net (i) RCILD (j) STAR-Net (k) STAR-Net-S

Real-world

- Classification



(a) Ground truth (b) Noisy (c) BM3D (d) BM4D (e) LLRT (f) LRTDTV

(g) NGMeet (h) HSI-SDeCNN (i) SMDS-Net (j) RCILD (k) STAR-Net (l) STAR-Net-S

Discussion

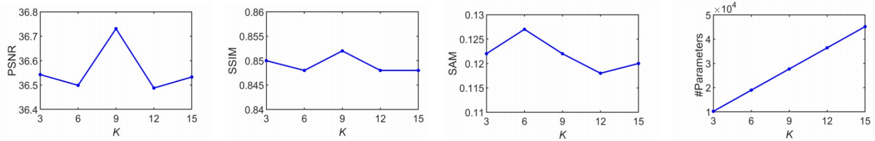
- Number of parameters

Methods	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
#Parameters	1,892,100	**5,103**	2,892,288	**27,702**	28,487

- Average runtime

Methods	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
Time	**11.027**	293.606	**30.706**	233.106	238.366

- Impact of unrolling iterations



(a) PSNR (b) SSIM (c) SAM (d) #Parameters

Outline

Conclusion

- What have we done?
  - How to characterize priors? $\Rightarrow$ Tensor + non-local self-similarity
  - How to develop algorithms? $\Rightarrow$ ADMM + deep unrolling

Dataset	Index	Noisy	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
ICVL	PSNR ↑	18.402	34.342	37.481	35.838	**39.853**	**39.963**
	SSIM ↑	0.042	0.893	0.907	**0.951**	**0.956**	**0.956**
	SAM ↓	0.779	0.134	0.066	0.092	**0.050**	**0.047**
Deep unrolling		-	×	✓	×	✓	✓
Non-local self-similarity		-	×	×	×	✓	✓

Thank you for your attention!

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References

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