第六章 学习最优化

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https://xianchaoxiu.github.io

目录

图像去噪

- 给定图像 $\mathbf{x} \in \mathbb{R}^d$, 考虑数学模型

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) + \gamma g(\mathbf{x})$$

  - $f(\mathbf{x})$ 是数据拟合项, $g(\mathbf{x})$ 是正则项
  - $\gamma \geq 0$ 是正则化参数

- 引入 $\mathbf{x} = \mathbf{y}$, 得到

$$\min_{\mathbf{x},\mathbf{y}} \quad f(\mathbf{y}) + \gamma g(\mathbf{x}) \quad \text{s.t.} \quad \mathbf{x} = \mathbf{y}$$

- 增广拉格朗日函数为

$$\mathcal{L}_\alpha(\mathbf{x}, \mathbf{y}, \mathbf{u}) = f(\mathbf{y}) + \gamma g(\mathbf{x}) + \mathbf{u}^\top(\mathbf{x} - \mathbf{y}) + \frac{1}{2}\|\mathbf{x} - \mathbf{y}\|^2$$

图像去噪

- 令 $\sigma^2 = \alpha\gamma$, 交替方向乘子算法

$$
\begin{aligned}
\mathbf{x}^{k+1} &= \arg\min_{\mathbf{x}} \ \{\sigma^2 g(\mathbf{x}) + \frac{1}{2}\|\mathbf{x} - (\mathbf{y}^k - \mathbf{u}^k)\|^2\} \\
\mathbf{y}^{k+1} &= \arg\min_{\mathbf{y}} \ \{\alpha f(\mathbf{y}) + \frac{1}{2}\|\mathbf{y} - (\mathbf{x}^{k+1} + \mathbf{u}^k)\|^2\} \\
\mathbf{u}^{k+1} &= \mathbf{u}^k + \mathbf{x}^{k+1} - \mathbf{y}^{k+1}
\end{aligned}
$$

- 具体形式为

$$
\begin{aligned}
\mathbf{x}^{k+1} &= \mathrm{prox}_{\sigma^2 g}(\mathbf{y}^k - \mathbf{u}^k) \quad \Rightarrow \quad \text{图像去噪} \\
\mathbf{y}^{k+1} &= \mathrm{prox}_{\alpha f}(\mathbf{x}^{k+1} + \mathbf{u}^k) \quad \Rightarrow \quad \text{与观测图像保持一致} \\
\mathbf{u}^{k+1} &= \mathbf{u}^k + \mathbf{x}^{k+1} - \mathbf{y}^{k+1}
\end{aligned}
$$

图像去噪

- **如何选择 $g(\mathbf{x})$**

∼1970	Energy regularization	$\|\mathbf{x}\|_2^2$
1975–1985	Spatial smoothness	$\|\mathbf{L}\mathbf{x}\|_2^2$ or $\|\mathbf{D}_v\mathbf{x}\|_2^2 + \|\mathbf{D}_h\mathbf{x}\|_2^2$
1980–1985	Optimally learned transform	$\|\mathbf{T}\mathbf{x}\|_2^2 = \mathbf{x}^T\mathbf{R}^{-1}\mathbf{x}$ where $\mathbf{T}/\mathbf{R}$ is learned via PCA
1980–1990	Weighted smoothness	$\|\mathbf{L}\mathbf{x}\|_{\mathbf{W}}^2$
1990–2000	Robust statistics	$\mathbf{1}^T\mu\{\mathbf{L}\mathbf{x}\}$ *e.g.*, Hubber-Markov
1992–2005	TV	$\int_{v\in\Omega}\|\nabla\mathbf{x}(v)\|dv$ or $\mathbf{1}^T\sqrt{\|\mathbf{D}_v\mathbf{x}\|^2+\|\mathbf{D}_h\mathbf{x}\|^2}$
1987–2005	Other PDE-based options	$\int_{v\in\Omega} g\left[\nabla\mathbf{x}(v), \nabla^2\mathbf{x}(v)\right] dv$
2005-2009	Field-of-experts	$\sum_k \lambda_k \mathbf{1}^T\mu_k\{\mathbf{L}_k\mathbf{x}\}$
1993–2005	Wavelet sparsity	$\|\mathbf{W}\mathbf{x}\|_1$
2000–2010	Self-similarity	$\sum_k\sum_{j\in\Omega(k)} d\{\mathbf{R}_k\mathbf{x}, \mathbf{R}_j\mathbf{x}\}$
2002–2012	Sparsity methods	$\|\alpha\|_0\ s.t.\ \mathbf{x}=\mathbf{D}\alpha$
2010–2017	Low-rank assumption	$\sum_k\|\mathbf{X}_{\Omega(k)}\|_*$

图像去噪

- 全变差（Total Variation, TV）去噪方法

$$\min_{\mathbf{x}} \quad \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|^2 + \lambda\|\mathbf{x}\|_{\mathrm{TV}}$$

其中 $\|\mathbf{x}\|_{\mathrm{TV}} = \sum |(\nabla \mathbf{x})_i|$，可取

$$\sqrt{(\mathbf{x}_{i+1}-\mathbf{x}_i)^2 + (\mathbf{x}_{i+n}-\mathbf{x}_i)^2} \quad 或 \quad |\mathbf{x}_{i+1}-\mathbf{x}_i| + |\mathbf{x}_{i+n}-\mathbf{x}_i|$$

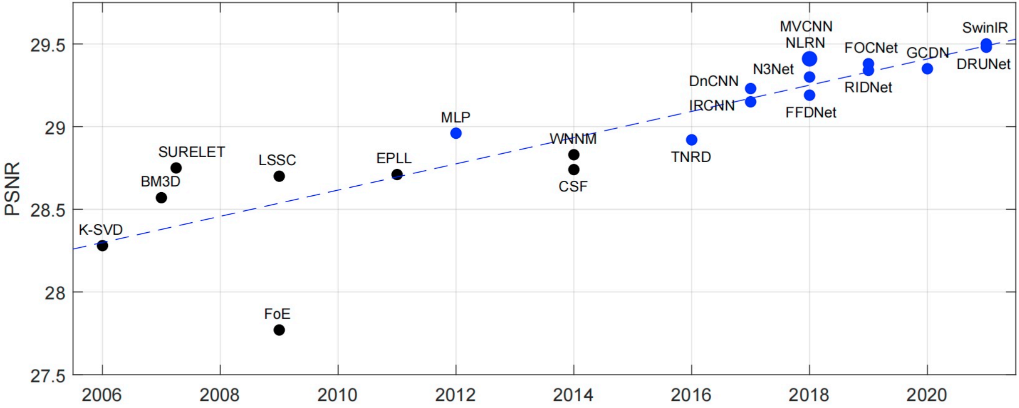
- 小波稀疏（Wavelet Sparsity）去噪方法

$$\min_{\mathbf{x}} \quad \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|^2 + \lambda\|W\mathbf{x}\|_1$$

$$\min_{\mathbf{x}} \quad \frac{1}{2}\|\mathbf{x}-\mathbf{y}\|^2 + \|\lambda \odot W\mathbf{x}\|_1$$

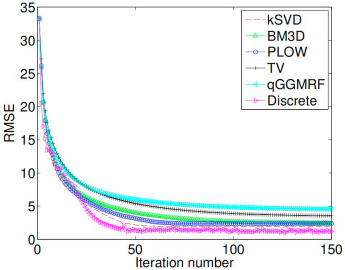
图像去噪

- 传统方法与深度学习方法



代表性工作

- 能否利用成熟的去噪器
  - 模型 + 数据，优势互补
  - 当数据不足时，无法进行端到端训练
- Plug-and-Play Priors for Model Based Reconstruction, 2013
- https://github.com/svvenkatakrishnan/plug-and-play-priors



Algorithm	RMSE	β
K-SVD [4]	2.13	4.32
BM3D [5]	2.46	1.39
PLOW [7]	2.35	1.50
TV [21]	3.55	0.47
q-GGMRF [22]	4.58	0.28
Discrete Recon [23]	1.20	1.00

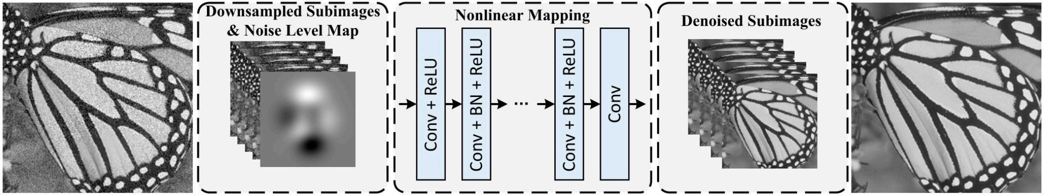
代表性工作: FFDNet

- Toward a Fast and Flexible Solution for CNN-Based Image Denoising, 2018
- https://github.com/cszn/FFDNet

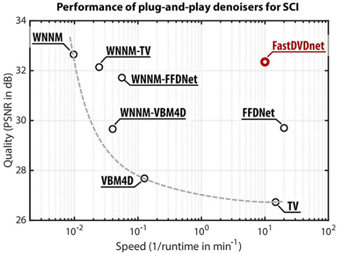
$$\min_{\mathbf{x}} \quad \frac{1}{2\sigma^2}\|\mathbf{x}-\mathbf{y}\|^2 + \lambda\Phi(\mathbf{x})$$

$$\Downarrow$$

$$\mathbf{x} = \mathcal{F}(\mathbf{y}, \sigma; \Theta)$$



- Plug-and-Play Algorithms for Video Snapshot Compressive Imaging, 2022
- https://github.com/liuyang12/PnP-SCI



PnP-ADMM 算法

- 即插即用 （Plug-and-Play, PnP）框架

$$\mathbf{x}^{k+1} = \mathrm{prox}_{\sigma^2 g}(\mathbf{y}^k - \mathbf{u}^k) \quad \Rightarrow \quad \mathbf{x}^{k+1} = H_\sigma(\mathbf{y}^k - \mathbf{u}^k)$$

- PnP-ADMM 迭代格式

$$\begin{aligned}
\mathbf{x}^{k+1} &= H_\sigma(\mathbf{y}^k - \mathbf{u}^k) \\
\mathbf{y}^{k+1} &= \mathrm{prox}_{\alpha f}(\mathbf{x}^{k+1} + \mathbf{u}^k) \\
\mathbf{u}^{k+1} &= \mathbf{u}^k + \mathbf{x}^{k+1} - \mathbf{y}^{k+1}
\end{aligned}$$

- 称 PnP-ADMM 为不动点迭代，如果 $(\mathbf{x}^*, \mathbf{u}^*)$ 满足

$$\begin{aligned}
\mathbf{x}^* &= H_\sigma(\mathbf{x}^* - \mathbf{u}^*) \\
\mathbf{x}^* &= \mathrm{prox}_{\alpha f}(\mathbf{x}^* + \mathbf{u}^*)
\end{aligned}$$

PnP-DRS 算法

- PnP-Douglas–Rachford Splitting (DRS) 迭代格式

$$
\begin{aligned}
\mathbf{x}^{k+1/2} &= \mathrm{prox}_{\alpha f}(\mathbf{z}^k) \\
\mathbf{x}^{k+1} &= H_\sigma(2\mathbf{x}^{k+1/2} - \mathbf{z}^k) \\
\mathbf{z}^{k+1} &= \mathbf{z}^k + \mathbf{x}^{k+1} - \mathbf{x}^{k+1/2}
\end{aligned}
$$

- 于是 PnP-DRS 的迭代可以表示为

$$
\begin{aligned}
\mathbf{z}^{k+1} &= T(\mathbf{z}^k) \\
T &= \frac{1}{2}I + \frac{1}{2}(2H_\sigma - I)(2\mathrm{prox}_{\alpha f} - I)
\end{aligned}
$$

- PnP-DRS 与 PnP-ADMM 等价

PnP-DRS 收敛性

- **假设** 因 σ 控制去噪的强度，H_σ 在较小的 σ 时接近恒等式。于是可以假设对于所有 $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$，存在 $\epsilon \geq 0$ 使得 $H_\sigma : \mathbb{R}^d \to \mathbb{R}^d$ 满足

$$\|(H_\sigma - I)(\mathbf{x}) - (H_\sigma - I)(\mathbf{y})\| \leq \epsilon \|\mathbf{x} - \mathbf{y}\|$$

- **定理** 假设 H_σ 满足上述条件，且 f 是 μ-强凸、可微函数。那么对于所有 $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$，存在 $\epsilon \geq 0$ 使得

$$T = \frac{1}{2} I + \frac{1}{2}(2H_\sigma - I)(2\mathrm{prox}_{\alpha f} - I)$$

满足

$$\|T(\mathbf{x}) - T(\mathbf{y})\| \leq \frac{1 + \epsilon + \epsilon\alpha\mu + 2\epsilon^2\alpha\mu}{1 + \alpha\mu + 2\epsilon\alpha\mu} \|\mathbf{x} - \mathbf{y}\|$$

若取 $\frac{\epsilon}{(1+\epsilon-2\epsilon^2)\mu} < \alpha$, $\epsilon < 1$，则 T 是**收缩算子**，即 PnP-DRS 算法收敛

PnP-ADMM 收敛性

- **定理** 假设 H_σ 关于 $\epsilon \in [0,1)$ 满足假设条件，且 f 是 μ-强凸函数。那么 PnP-ADMM 算法收敛只需要

$$\frac{\epsilon}{(1+\epsilon-2\epsilon^2)\mu} < \alpha$$

- 注意到 H_σ 是去噪器，R 是残差，I 是恒等变化

$$(I - H_\sigma)(\mathbf{y}) = \mathbf{y} - H_\sigma(\mathbf{y}) = R(\mathbf{y})$$

于是假设条件等价于限制残差 R 的利普希斯常数

Plug-and-play methods provably converge with properly trained denoisers

E Ryu, J Liu, S Wang, X Chen… - … on Machine Learning, 2019 - proceedings.mlr.press

Abstract Plug-and-play (PnP) is a non-convex framework that integrates modern denoising priors, such as BM3D or deep learning-based denoisers, into ADMM or other proximal

案例

IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT, VOL. 74, 2025 5503014

Tensor Low-Rank Approximation via Plug-and-Play Priors for Anomaly Detection in Remote Sensing Images

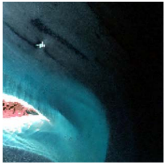
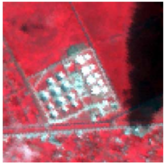
Jingjing Liu, Manlong Feng, Xianchao Xiu, *Member, IEEE*, Xiaoyang Zeng, *Senior Member, IEEE*, and Jianhua Zhang, *Senior Member, IEEE*

***Abstract*— Optical remote sensing images (RSIs) have received widespread attention in fields such as agricultural monitoring, mineral exploration, and military defense. However, the detection performance will be seriously degraded when interfered with by noise. To overcome this issue, we first present a novel method called tensor low-rank approximation (TLRA), which leverages the weighted tensor nuclear norm (WTNN) to exploit the spectral overall structure, introduces a new tensor sparse $\ell_{F,0}$ term to**

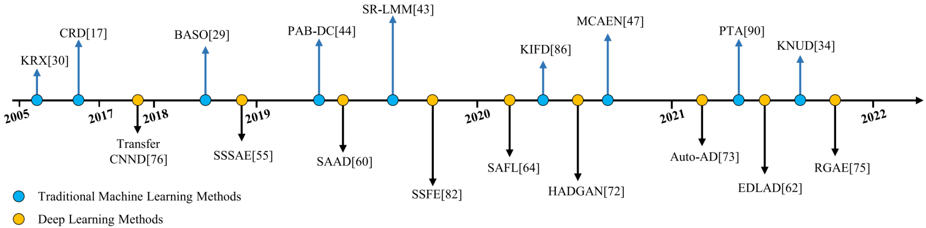
I. Introduction

ANOMALY detection is a prominent field of study in the realm of optical remote sensing images (RSIs) [1], [2], [3], widely used in environmental monitoring [4], agricultural management [5], military reconnaissance [6], and other scenarios. The target of anomaly detection is to detect abnormal,

- 高光谱图像异常检测（Hyperspectral Anomaly Detection, HAD）

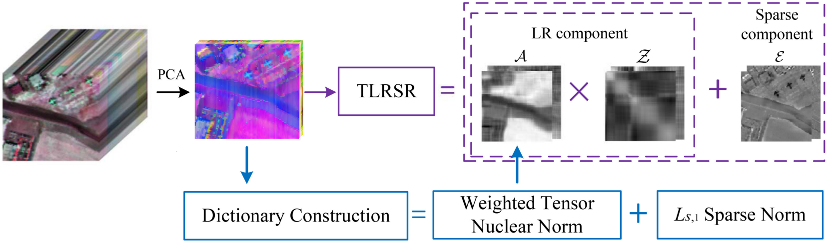





- 代表性方法

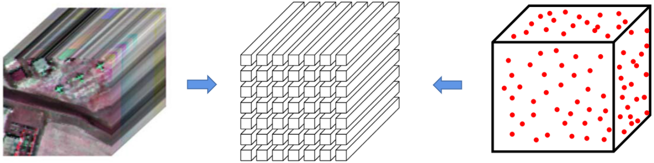


- Learning Tensor Low-Rank Representation for Hyperspectral Anomaly Detection, 2023

$$\min_{\mathcal{Z},\mathcal{E}} \|\mathcal{Z}\|_{\mathrm{WTNN}} + \lambda\|\mathcal{E}\|_1$$
$$\text{s.t. } \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E}$$



■ 如何刻画张量稀疏性?



□ 元素稀疏

$$\|\mathcal{E}\|_1 = \sum_{i=1}^{h}\sum_{j=1}^{v}\sum_{k=1}^{z}|\mathcal{E}(i,j,k)| \qquad \Rightarrow \qquad \|\mathcal{E}\|_0 = \sharp\{(i,j,k) \mid |\mathcal{E}(i,j,k)| \neq 0\}$$

□ 结构稀疏

$$\|\mathcal{E}\|_{\mathrm{F},0} = \sharp\{(i,j) \mid \|\mathcal{E}(i,j,:)\|_2 \neq 0\}$$

- Plug-and-Play Tensor Low-Rank Approximation (PnP-TLRA)

$$\begin{aligned}\min_{\mathcal{Z},\mathcal{E}}\ & \|\mathcal{Z}\|_{\mathrm{WTNN}} + \lambda\|\mathcal{E}\|_1 \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E}\end{aligned}$$

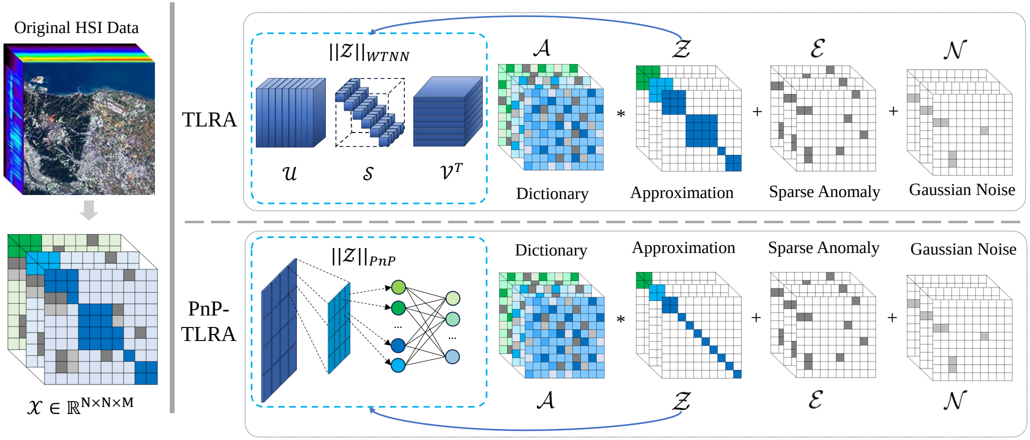
$$\Downarrow$$

$$\text{(TLRA)} \quad \begin{aligned}\min_{\mathcal{Z},\mathcal{E},\mathcal{N}}\ & {\color{blue}\|\mathcal{Z}\|_{\mathrm{WTNN}} + \lambda\|\mathcal{E}\|_{\mathrm{F},0} + \mu\|\mathcal{N}\|_{\mathrm{F}}^2} \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} + \mathcal{N}\end{aligned}$$

$$\Downarrow$$

$$\text{(PnP-TLRA)} \quad \begin{aligned}\min_{\mathcal{Z},\mathcal{E},\mathcal{N}}\ & {\color{red}\phi(\mathcal{Z}) + \lambda\|\mathcal{E}\|_{\mathrm{F},0} + \mu\|\mathcal{N}\|_{\mathrm{F}}^2} \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} + \mathcal{N}\end{aligned}$$

数学建模



Original HSI Data
$\mathcal{X} \in \mathbb{R}^{N\times N\times M}$
TLRA
$||\mathcal{Z}||_{WTNN}$
$\mathcal{U}$
$\mathcal{S}$
$\mathcal{V}^T$
$\mathcal{A}$
$\mathcal{Z}$
$\mathcal{E}$
$\mathcal{N}$
Dictionary
Approximation
Sparse Anomaly
Gaussian Noise
PnP-
TLRA
$||\mathcal{Z}||_{PnP}$
Dictionary
Approximation
Sparse Anomaly
Gaussian Noise
$\mathcal{A}$
$\mathcal{Z}$
$\mathcal{E}$
$\mathcal{N}$

优化算法

- PnP-ADMM 算法

$$L_\beta(\mathcal{Z},\mathcal{E},\mathcal{N},\mathcal{Y}) = \phi(\mathcal{Z}) + \lambda\|\mathcal{E}\|_{\mathrm{F},0} + \mu\|\mathcal{N}\|_{\mathrm{F}}^2 - \langle \mathcal{Y}, \mathcal{X} - \mathcal{A} * \mathcal{Z} - \mathcal{E} - \mathcal{N}\rangle + \frac{\beta}{2}\|\mathcal{X} - \mathcal{A} * \mathcal{Z} - \mathcal{E} - \mathcal{N}\|_{\mathrm{F}}^2$$

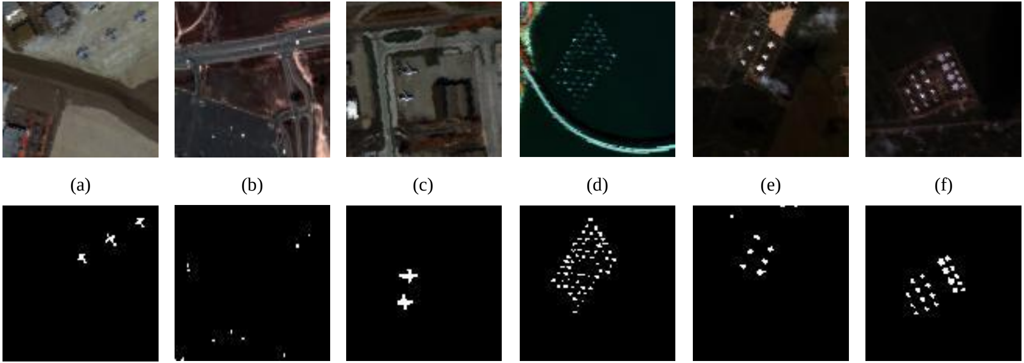
- 迭代过程
  - $\mathcal{Z}^{k+1} = \arg\min_{\mathcal{Z}} \ \phi(\mathcal{Z}) + \frac{\beta}{2}\|\mathcal{X} - \mathcal{A} * \mathcal{Z} - \mathcal{E}^k - \mathcal{N}^k - \mathcal{Y}^k/\beta\|_{\mathrm{F}}^2$
  - $\mathcal{E}^{k+1} = \arg\min_{\mathcal{E}} \ \lambda\|\mathcal{E}\|_{\mathrm{F},0} + \frac{\beta}{2}\|\mathcal{X} - \mathcal{A} * \mathcal{Z}^{k+1} - \mathcal{E} - \mathcal{N}^k - \mathcal{Y}^k/\beta\|_{\mathrm{F}}^2$
  - $\mathcal{N}^{k+1} = \arg\min_{\mathcal{N}} \ \mu\|\mathcal{N}\|_{\mathrm{F}}^2 + \frac{\beta}{2}\|\mathcal{X} - \mathcal{A} * \mathcal{Z}^{k+1} - \mathcal{E}^{k+1} - \mathcal{N} - \mathcal{Y}^k/\beta\|_{\mathrm{F}}^2$
  - $\mathcal{Y}^{k+1} = \mathcal{Y}^k - \beta(\mathcal{X} - \mathcal{A} * \mathcal{Z}^{k+1} - \mathcal{E}^{k+1} - \mathcal{N}^{k+1})$
- 算法产生的点列 $\{(\mathcal{Z}^k,\mathcal{E}^k,\mathcal{N}^k,\mathcal{Y}^k)\}$ 收敛到 TLRA 的稳定点, 在一定的条件下 PnP-TLRA 也是收敛的

数值实验

- GRXD: Hyperspectral Remote Sensing, 2012
- RPCA-RX: Low-Rank and Sparse Matrix Decomposition-Based Anomaly Detection for Hyperspectral Imagery, 2019
- LSMAD: A Low-Rank and Sparse Matrix Decomposition-Based Mahalanobis Distance Method for Hyperspectral Anomaly Detection, 2015
- GTVLRR: Graph and Total Variation Regularized Low-Rank Representation for Hyperspectral Anomaly Detection, 2020
- GAED: Hyperspectral Anomaly Detection with Guided Autoencoder, 2022
- GNLTR: Generalized Nonconvex Low-Rank Tensor Representation for Hyperspectral Anomaly Detection, 2023
- ELRSF-SP: Hyperspectral Anomaly Detection via Enhanced Low-Rank and Smoothness Fusion Regularization Plus Saliency Prior, 2024
- TPCA: A Preprocessing Method for Hyperspectral Target Detection Based on Tensor Principal Component Analysis, 2018
- PTA: Prior-Based Tensor Approximation for Anomaly Detection in Hyperspectral Imagery, 2022
- PCA-TLRSR: Learning Tensor Low-Rank Representation for Hyperspectral Anomaly Detection, 2023

数值实验

- 数据集 (a) San Diego (b) HYDICE (c) ABU-airport (d) ABU-beach (e)-(f) ABU-urban

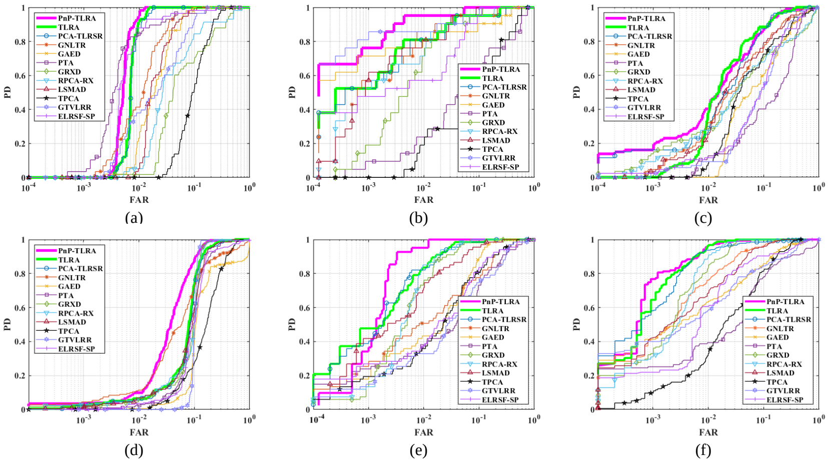


(a) (b) (c) (d) (e) (f)

(g) (h) (i) (j) (k) (l)

数值实验

- 接收者操作特征曲线（Receiver Operating Characteristic, ROC)



(a) (b) (c)

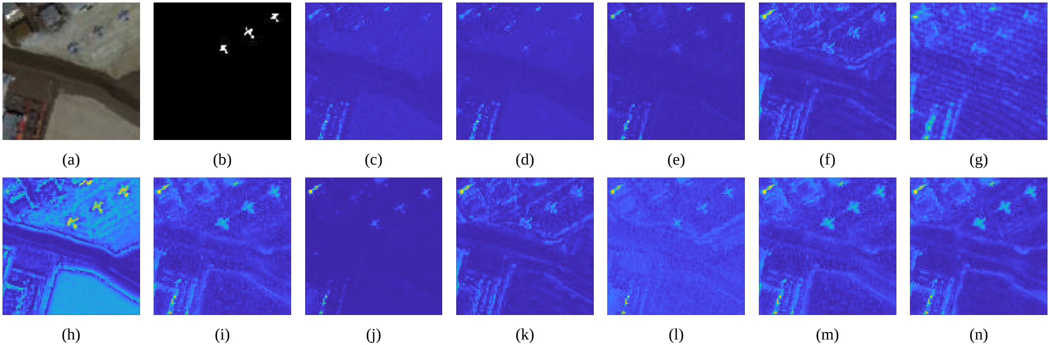
(d) (e) (f)

数值实验

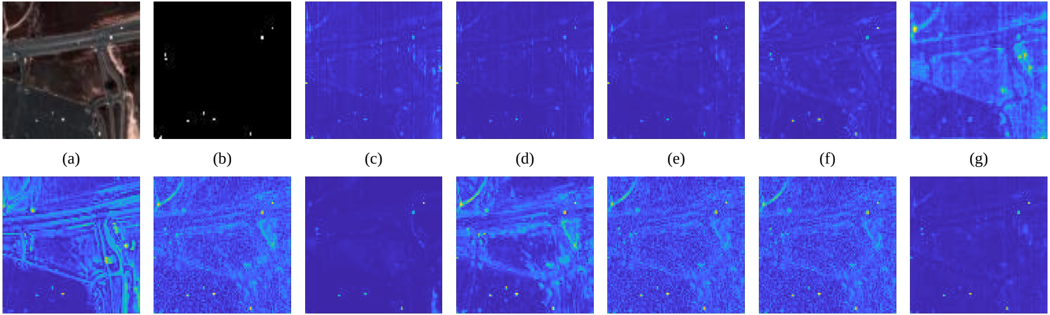
■ 曲线下面积（Area Under the Curve, AUC）

	GRXD	RPCA-RX	LSMAD	GTVLRR	TPCA	PTA	PCA-TLRSR	GAED	GNLTR	ELRSF-SP	TLRA	PnP-TLRA
SD	0.8886	0.9165	0.9457	0.9648	0.8849	0.9868	0.9923	0.9889	0.9838	0.9878	0.9927	**0.9942**
HYDICE	0.9857	0.9842	0.9906	0.9918	0.8242	0.8396	0.9802	0.9639	0.9859	0.9768	0.9840	**0.9968**
ABU-airport-1	0.8221	0.8089	0.8341	0.8957	0.8023	0.7698	0.9291	0.8106	0.9293	0.8221	**0.9331**	0.9303
ABU-airport-2	0.8403	0.8431	0.9192	0.8911	0.8891	0.8995	0.9345	0.9272	0.9242	0.8891	0.9409	**0.9523**
ABU-airport-3	0.9288	0.9275	0.9398	0.9287	0.9297	0.7369	0.9233	0.8638	0.9330	0.9209	0.9379	**0.9380**
ABU-airport-4	0.9526	0.9627	0.9864	0.9776	0.9432	0.9890	0.9914	0.9654	0.9606	0.9876	**0.9932**	0.9918
ABU-beach-1	0.9804	0.9761	0.9778	0.9703	0.9860	0.9682	0.9831	0.9223	0.9638	0.9780	0.9868	**0.9895**
ABU-beach-2	0.9106	0.9097	0.9056	0.9348	0.8061	0.8862	0.9331	0.5061	0.9472	0.9097	**0.9580**	0.9533
ABU-beach-3	**0.9998**	0.9995	0.9996	0.9866	0.9982	0.9483	0.9994	0.9891	0.9928	0.9945	0.9994	0.9996
ABU-beach-4	0.9538	0.9599	0.9349	0.9803	0.9338	0.9000	0.9755	0.8514	0.9044	0.9698	0.9899	**0.9902**
ABU-urban-1	0.9907	0.9922	0.9818	0.8742	0.9390	0.8940	0.9923	0.9409	0.9325	0.9907	0.9934	**0.9951**
ABU-urban-2	0.9946	0.9957	0.9849	0.8628	0.9409	0.9701	0.9928	0.9958	0.9874	0.9946	0.9983	**0.9994**
ABU-urban-3	0.9513	0.9577	0.9633	0.9365	0.8224	0.9090	0.9832	0.9725	0.9667	0.9513	**0.9888**	0.9855
ABU-urban-4	0.9887	0.9871	0.9809	0.9205	0.9835	0.9937	0.9857	0.9556	0.9931	0.9909	0.9883	**0.9950**
ABU-urban-5	0.9690	0.9658	0.9610	0.9347	0.9370	0.8693	0.9811	0.9148	0.9297	0.9658	0.9821	**0.9837**
Average	0.9438	0.9458	0.9537	0.9367	0.9080	0.9040	0.9718	0.9046	0.9556	0.9553	0.9778	**0.9796**

■ San Diego 数据集比较



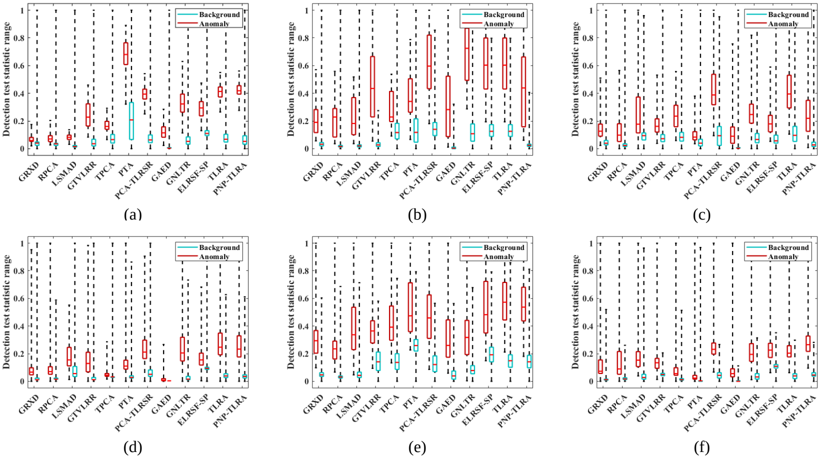
HYDICE 数据集比较



(a) (b) (c) (d) (e) (f) (g)

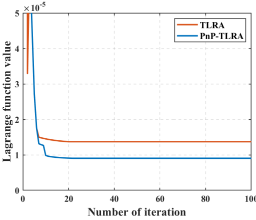
(h) (i) (j) (k) (l) (m) (n)

■ 统计可分性比较

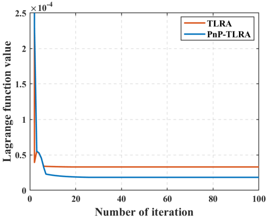

Detection test statistic range
0
0.2
0.4
0.6
0.8
1
Background
Anomaly
GRXD
RPCA
LSMAD
GTVLRR
TPCA
PTA
PCA-TLRSR
GAED
GNLTR
ELRSF-SP
TLRA
PNP-TLRA
(a)
(b)
(c)
(d)
(e)
(f)

数值实验

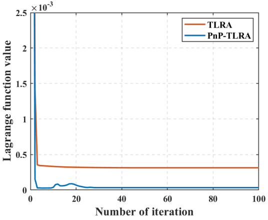
■ 目标函数损失



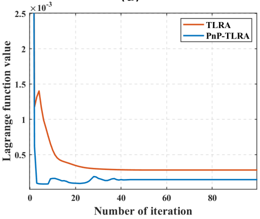
(a)



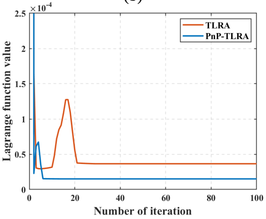
(b)



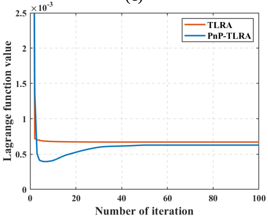
(c)



(d)



(e)



(f)

数值实验

去噪模块比较

Datasets	BM3D		DnCNN		LeNet		FFDNet	
	AUC	Time	AUC	Time	AUC	Time	AUC	Time
San Diego	0.9911	17.74	0.9896	15.65	0.9902	17.72	0.9942	13.21
HYDICE	0.9946	21.02	0.9883	22.15	0.9915	16.63	0.9968	15.43
ABU-airport-2	0.9436	13.93	0.9407	11.98	0.9344	12.48	0.9523	8.69
ABU-beach-2	0.9491	17.66	0.9501	19.58	0.9472	15.4	0.9533	12.68
ABU-urban-2	0.9955	14.84	0.9951	12.43	0.9986	13.95	0.9994	10.21

运行时间比较

Datasets	TPCA	PTA	PCA-TLRSR	TLRA	PnP-TLRA
San Diego	34.66	26.37	10.99	**5.89**	<u>10.63</u>
HYDICE	25.39	22.76	<u>9.31</u>	**4.89**	24.49
ABU-airport	41.63	29.21	11.32	**6.12**	<u>9.71</u>
ABU-beach	29.04	38.24	15.61	**10.86**	<u>12.31</u>
ABU-urban	36.95	28.01	17.35	**8.38**	<u>11.82</u>

小结

- **如何选择去噪器**
- **如何选择模型参数**
  - Tuning-Free Plug-and-Play Proximal Algorithm for Inverse Imaging Problems, 2020
- **如何改善收敛性**
  - Enhanced Convergent PnP Algorithms for Image Restoration, 2021
  - Wasserstein-based Projections with Applications to Inverse Problems, 2022
- **综述**
  - Image Denoising: The Deep Learning Revolution and Beyond, 2023
  - Learned Reconstruction Methods With Convergence Guarantees, 2023
  - Plug-and-Play Methods for Integrating Physical and Learned Models in Computational Imaging, 2023
  - Learning to Optimize: A Tutorial for Continuous and Mixed-Integer Optimization, 2024

目录

压缩感知

- 压缩感知通常求解以下优化问题

$$\min_{x} \ \frac{1}{2}\|\Phi x - y\|_2^2 + \lambda\|\Psi x\|_1 \tag{1}$$

- 迭代收缩阈值算法（Iterative Soft Thresholding Algorithm, ISTA）

De-noising by soft-thresholding

DL Donoho - IEEE transactions on information theory, 2002 - ieeexplore.ieee.org

… We propose here a formal interpretation of the term “denoising” and show how wavelet transforms may be used to optimally “de-noise” in this interpretation. Moreover, this “denoising”

☆ 保存 99 引用 被引用次数：15994 相关文章 所有 12 个版本

ISTA

- 梯度步

$$r^{(k)} = x^{(k-1)} - \rho\Phi^\top(\Phi x^{(k-1)} - y)$$

- 收缩阈值步

$$x^{(k)} = \arg\min_x \frac{1}{2}\|x - r^{(k)}\|_2^2 + \lambda\|\Psi x\|_1$$

$$\Downarrow$$

$$\begin{aligned} x^{(k)} &= \mathrm{Prox}_{\lambda\phi}(r^{(k)}) \\ &= \mathrm{sign}(r^{(k)}) \cdot \max(|r^{(k)}|\lambda, 0) \end{aligned}$$

- 收敛速度比较慢
- 性能很大程度上取决于超参数的选择

LISTA

- 深度展开网络（Deep Unfolding Networks, DUNs）
- LISTA 将 ISTA 的每一步迭代模拟为神经网络的一层

$$r^{(k)} = x^{(k-1)} - \rho\Phi^\top(\Phi x^{(k-1)} - y)$$

$$\Downarrow$$

$$x^{(k+1)} = h_\theta(x^{(k-1)} - \rho\Phi^\top(\Phi x^{(k-1)} - y))$$

Learning fast approximations of sparse coding

K Gregor, Y LeCun

Proceedings of the 27th international conference on international conference …, 2010 • dl.acm.org

> In Sparse Coding (SC), input vectors are reconstructed using a sparse linear combination of basis vectors. SC has become a popular method for extracting features from data. For a given input, SC minimizes a quadratic reconstruction error with an L_1 penalty term on

ISTA-Net

- ISTA-Net = LISTA + CNN

$$\min_x \frac{1}{2}\|\Phi x - y\|_2^2 + \lambda\|\Psi x\|_1$$

$$\Downarrow$$

$$\min_x \frac{1}{2}\|\Phi x - y\|_2^2 + \lambda\|F(x)\|_1$$

ISTA-Net: Interpretable optimization-inspired deep network for image compressive sensing

J Zhang, B Ghanem - … of the IEEE conference on computer …, 2018 - openaccess.thecvf.com

… el structured deep network, dubbed ISTA-Net, which is inspired … All the parameters in ISTA-Net (eg. nonlinear transforms, … version of ISTA-Net in the residual domain, dubbed ISTA-Net+, …

☆ 保存 99 引用 被引用次数：1555 相关文章 所有 11 个版本 »

案例

Pattern Recognition 172 (2026) 112496

ELSEVIER

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/pr

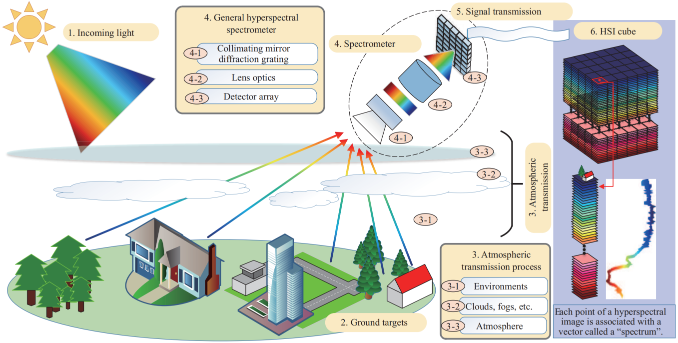
STAR-Net: an interpretable model-aided network for remote sensing image denoising

Jingjing Liu [a,b], Jiashun Jin [a], Xianchao Xiu [c,*], Jianhua Zhang [a], Wanquan Liu [d]

[a] *School of Microelectronics, Shanghai Collaborative Innovation Center for Intelligent Sensing Chip Technology, Shanghai University, Shanghai, 200444, China*
[b] *State Key Laboratory of Integrated Chips and Systems, Fudan University, Shanghai, 201203, China*
[c] *School of Mechatronic Engineering and Automation, Shanghai University, Shanghai, 200444, China*
[d] *School of Intelligent Systems Engineering, Sun Yat-Sen University, Shenzhen, 510275, China*

高光谱图像去噪

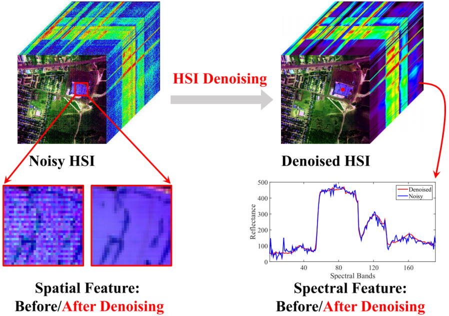
- 为什么选择高光谱图像



- 多样任务: 去噪, 分类, 异常检测, 融合, 解混
- https://github.com/xianchaoxiu/Hyperspectral-Imaging

高光谱图像去噪

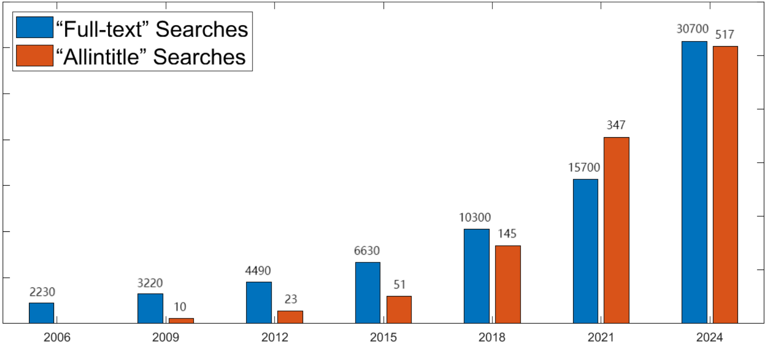
- 高光谱图像去噪/高光谱图像恢复



- 基于模型: 全变差, 稀疏编码, 低秩表示
- 数据驱动: 卷积神经网络, 混合网络, 无监督网络

高光谱图像去噪

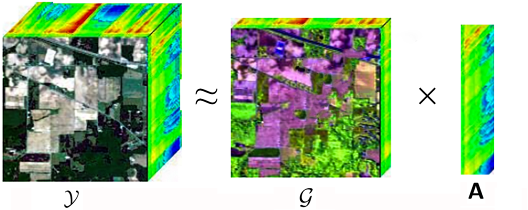
- Google Scholar 中关于“tensor hyperspectral”的统计



- 出色的数据表示，多样的张量分解，较低的计算复杂度

子空间张量表示框架

$$(\mathrm{P}) \quad \min_{\mathcal{G},\mathbf{A}} \quad \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda\Omega(\mathcal{G})$$
$$\text{s.t.} \quad \mathbf{A}^\top\mathbf{A} = \mathbf{I}$$



- Liu-Li-Wang-Tao-Du-Chanussot, SCIS, 2023
- Wang-Hong-Han-Li-Yao-Gao, IEEE GRSM, 2023

数学建模

- Xiong-Zhou-Tao-Lu-Zhou-Qian, IEEE TIP, 2022

$$\begin{aligned}
\min_{\mathcal{G},\mathcal{B}_i,\mathbf{A}} \quad & \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}\|_{\mathrm{F}}^2 + \lambda \sum_i (\phi(\mathcal{G},\mathcal{B}_i) + \gamma_1 \|\mathcal{B}_i\|_1) \\
\text{s.t.} \quad & \mathbf{A}^\top \mathbf{A} = \mathbf{I}
\end{aligned}$$

- 多维度表示

$$\phi(\mathcal{G},\mathcal{B}_i) = \frac{1}{2}\|\mathcal{R}_i \mathcal{G} - \mathcal{B}_i \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3\|_{\mathrm{F}}^2$$

- 如何刻画先验信息?
- 如何开发快速算法?

数学建模

- 稀疏张量辅助表示 (Sparse tensor aided representation, STAR)

$$
\begin{aligned}
\text{(STAR)} \quad \min_{\mathcal{G},\mathcal{B}_i,\mathbf{A}} \quad & \frac{1}{2}\|\mathcal{Y}-\mathcal{G}\times_3\mathbf{A}\|_{\mathrm{F}}^2+\lambda\sum_i\left(\phi(\mathcal{G},\mathcal{B}_i)+\gamma_1\|\mathcal{B}_i\|_1+\gamma_2\|\mathcal{B}_i\|_*\right) \\
\text{s.t.} \quad & \mathbf{A}^\top\mathbf{A}=\mathbf{I}
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\text{(STAR-S)} \quad \min_{\mathcal{G},\mathcal{S},\mathcal{B}_i,\mathbf{A}} \quad & \frac{1}{2}\|\mathcal{Y}-\mathcal{G}\times_3\mathbf{A}-\mathcal{S}\|_{\mathrm{F}}^2+\mu\|\mathcal{S}\|_1 \\
& +\lambda\sum_i\left(\phi(\mathcal{G},\mathcal{B}_i)+\gamma_1\|\mathcal{B}_i\|_1+\gamma_2\|\mathcal{B}_i\|_*\right) \\
\text{s.t.} \quad & \mathbf{A}^\top\mathbf{A}=\mathbf{I}
\end{aligned}
$$

- ADMM 算法

$$
\begin{aligned}
\min_{\mathcal{G},\mathcal{B}_i,\mathcal{L}_i,\mathbf{A}} \quad & \frac{1}{2}\|\mathcal{Y}-\mathcal{G}\times_3\mathbf{A}\|_{\mathrm{F}}^2+\lambda\sum_i\left(\phi(\mathcal{G},\mathcal{B}_i)+\gamma_1\|\mathcal{B}_i\|_1+\gamma_2\|\mathcal{L}_i\|_*\right)\\
\text{s.t.} \quad & \mathbf{A}^\top\mathbf{A}=\mathbf{I},\ \mathcal{L}_i=\mathcal{B}_i
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
L_\beta(\mathcal{G},\mathcal{B}_i,\mathcal{L}_i,\mathbf{A},\mathcal{P}_i)=&\frac{1}{2}\|\mathcal{Y}-\mathcal{G}\times_3\mathbf{A}\|_{\mathrm{F}}^2+\langle\mathcal{P}_i,\mathcal{L}_i-\mathcal{B}_i\rangle\\
&+\lambda\sum_i\left(\phi(\mathcal{G},\mathcal{B}_i)+\gamma_1\|\mathcal{B}_i\|_1+\gamma_2\|\mathcal{L}_i\|_*\right)+\frac{\beta}{2}\|\mathcal{L}_i-\mathcal{B}_i\|_{\mathrm{F}}^2
\end{aligned}
$$

- 深度展开网络

- **计算 $\mathcal{G}$-子问题**

$$
\begin{aligned}
\mathcal{G}^{k+1} = &\arg\min_{\mathcal{G}} \ \frac{1}{2}\|\mathcal{Y} - \mathcal{G} \times_3 \mathbf{A}^k\|_{\mathrm{F}}^2 \\
&+ \frac{\lambda}{2}\sum_i \|\mathcal{R}_i\mathcal{G} - \mathcal{B}_i^k \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3\|_{\mathrm{F}}^2
\end{aligned}
$$

$$\Downarrow$$

$$\mathcal{G}^{k+1} = (\mathcal{I} + \lambda\sum_i \mathcal{R}_i^\top\mathcal{R}_i)^{-1}(\lambda\sum_i \mathcal{R}_i^\top\mathcal{B}_i^k \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3 + \mathcal{Y} \times_3 \mathbf{A}^{k\top})$$

$$\Downarrow$$

$$\mathcal{G}^{k+1} = \mathcal{E}_1 * \mathcal{E}_2$$

$$\Downarrow$$

$$\mathcal{G}^{k+1} = \mathrm{LargNet}(\mathcal{E}_1, \mathcal{E}_2)$$

优化算法

- 计算 $\mathcal{B}_i$-子问题

$$
\begin{aligned}
\mathcal{B}_i^{k+1} &= \arg\min_{\mathcal{B}_i} \frac{\lambda}{2}\|\mathcal{R}_i\mathcal{G}^{k+1} - \mathcal{B}_i \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3\|_{\mathrm{F}}^2 \\
&\quad + \frac{\beta}{2}\|\mathcal{L}_i^k - \mathcal{B}_i + \mathcal{P}_i^k/\beta\|_{\mathrm{F}}^2 + \lambda\gamma_1\|\mathcal{B}_i\|_1
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
\mathcal{B}_i^{k+1} &= \arg\min_{\mathcal{B}_i} \frac{1}{2}\|(\beta\mathcal{I} + \lambda\mathcal{I} \times_1 \mathbf{D}_1 \times_2 \mathbf{D}_2 \times_3 \mathbf{D}_3)\mathcal{B}_i \\
&\quad - (\lambda\mathcal{R}_i\mathcal{G}^{k+1} + \beta\mathcal{L}_i^k + \mathcal{P}_i^k)\|_{\mathrm{F}}^2 + \lambda\gamma_1\|\mathcal{B}_i\|_1
\end{aligned}
$$

$$\Downarrow$$

$$\mathcal{B}_i^{k+1} = \mathrm{Shrink}(\mathcal{F}_i, \lambda\gamma_1/\nu)$$

$$\Downarrow$$

$$\mathcal{B}_i^{k+1} = \mathrm{sgn}(\mathcal{F}_i)\mathrm{ReLU}(|\mathcal{F}_i| - \lambda\gamma_1/\nu)$$

优化算法

- **计算 A-子问题**

$$\mathbf{A}^{k+1} = \arg\min_{\mathbf{A}^\top\mathbf{A}=\mathbf{I}} \frac{1}{2}\|\mathcal{Y} - \mathcal{G}^{k+1} \times_3 \mathbf{A}\|_{\mathrm{F}}^2$$

$$\Downarrow$$

$$\mathbf{A}^{k+1} = \mathbf{U}\mathbf{V}^\top$$

$$\Downarrow$$

$$\mathbf{A}^{k+1} = \mathrm{LargNet}(\mathbf{U}, \mathbf{V}^\top)$$

- **计算 $\mathcal{P}_i$-子问题**

$$\mathcal{P}_i^{k+1} = \mathcal{P}_i^k + \beta(\mathcal{L}_i^{k+1} - \mathcal{B}_i^{k+1})$$

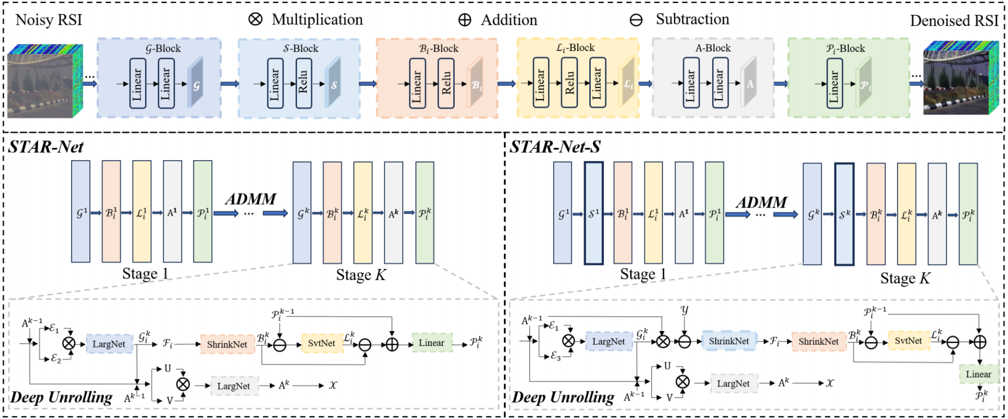
$$\Downarrow$$

$$\mathcal{P}_i^{k+1} = \mathrm{Linear}(\Theta_i)$$

优化算法

- **输入**: **数据** $\mathcal{Y}$, **参数** $\lambda, \beta, \gamma_1, \gamma_2, \nu$
- **初始化**: $(\mathcal{G}^0, \mathcal{B}_i^0, \mathcal{L}_i^0, \mathbf{A}^0, \mathcal{P}_i^0)$
- **当** $k = 1, \ldots, K$ **更新**
  - $\mathcal{G}^{k+1} = \mathrm{LargNet}(\mathcal{E}_1, \mathcal{E}_2)$
  - $\mathcal{B}_i^{k+1} = \mathrm{ShrinkNet}(\mathcal{F}_i, \lambda\gamma_1/\nu)$
  - $\mathcal{L}_i^{k+1} = \mathrm{SvtNet}(\mathcal{B}_i^{k+1} - \mathcal{P}_i^k/\beta, \lambda\gamma_2/\beta)$
  - $\mathbf{A}^{k+1} = \mathrm{LargNet}(\mathbf{U}, \mathbf{V}^\top)$
  - $\mathcal{P}_i^{k+1} = \mathrm{Linear}(\Theta_i)$
- **输出**: $\mathcal{X} = \mathcal{G}^{k+1} \times_3 \mathbf{A}^{k+1}$

优化算法



Noisy RSI
Multiplication
Addition
Subtraction
Denoised RSI
G-Block
S-Block
B_i-Block
L_i-Block
A-Block
P_i-Block
Linear
Relu
STAR-Net
STAR-Net-S
ADMM
Stage 1
Stage K
LargNet
ShrinkNet
SvtNet
Linear
Deep Unrolling

数值实验

- 对比方法
  - BM3D: Dabov-Katkovnik-Egiazarian, IEEE TIP, 2007
  - BM4D: Maggioni-Katkovnik-Egiazarian-Foi, IEEE TIP, 2012
  - LLRT: Chang-Yan-Zhong, CVPR, 2017
  - LRTDTV: Wang-Chen-Han-He, RS, 2017
  - NGMeet: He-Yao-Li-Yokoya-Zhao-Zhang-Zhang, IEEE TPAMI, 2022
  - HSI-SDeCNN: Maffei-Haut-Paoletti-Plaza, IEEE TGRS, 2020
  - SMDS-Net: Xiong-Zhou-Tao-Lu-Zhou-Qian, IEEE TIP, 2022
  - RCILD: Peng-Wang-Cao-Zhao-Yao-Zhang-Meng, IEEE TGRS, 2024
- 损失函数 $L = \|\mathrm{STAR\text{-}Net}(\mathcal{Y}) - \mathcal{X}\|_{\mathrm{F}}^2$
- 可训练参数 $\lambda, \beta, \mu, \gamma_1, \gamma_2, \mathbf{D}_1, \mathbf{D}_2, \mathbf{D}_3$

数值实验

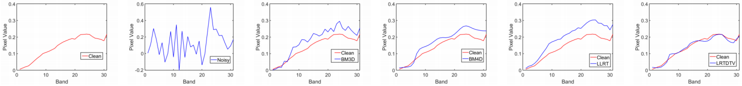
ICVL 数据集比较

σ	Index	Noisy	Model-based methods					Deep learning-based methods				
			BM3D	BM4D	LLRT	LRTDTV	NGMeet	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
10	PSNR $\uparrow$	29.018	37.310	42.987	39.810	43.882	42.383	41.519	46.371	42.458	**47.286**	**47.345**
	SSIM $\uparrow$	0.521	0.924	0.973	0.962	0.979	0.966	0.969	0.985	0.987	**0.988**	**0.989**
	SAM $\downarrow$	0.229	0.121	0.080	0.045	0.077	0.074	0.075	**0.028**	0.044	**0.025**	**0.025**
30	PSNR $\uparrow$	21.591	32.582	37.630	34.250	38.245	36.791	36.840	42.337	38.514	**42.435**	**42.500**
	SSIM $\uparrow$	0.146	0.846	0.930	0.921	0.877	0.915	0.926	**0.972**	**0.971**	**0.972**	**0.972**
	SAM $\downarrow$	0.535	0.208	0.142	0.084	0.149	0.139	0.124	0.040	0.067	**0.039**	**0.038**
50	PSNR $\uparrow$	18.402	29.982	35.242	32.067	33.618	34.399	34.342	37.481	35.838	**39.853**	**39.963**
	SSIM $\uparrow$	0.042	0.790	0.888	0.899	0.862	0.887	0.893	0.907	**0.951**	**0.956**	**0.956**
	SAM $\downarrow$	0.779	0.264	0.190	0.107	0.195	0.177	0.134	0.066	0.092	**0.050**	**0.047**
70	PSNR $\uparrow$	18.126	28.654	33.586	30.746	30.565	32.389	32.794	37.197	33.980	**37.342**	**38.237**
	SSIM $\uparrow$	0.038	0.742	0.844	0.852	0.762	0.858	0.855	0.923	**0.930**	**0.943**	**0.943**
	SAM $\downarrow$	0.897	0.310	0.231	0.214	0.304	0.217	0.186	0.066	0.132	**0.058**	**0.055**
Ave.	PSNR $\uparrow$	21.784	32.132	37.361	34.218	36.578	36.490	36.374	40.463	37.212	**41.729**	**42.011**
	SSIM $\uparrow$	0.187	0.826	0.909	0.909	0.870	0.907	0.911	0.943	**0.953**	**0.965**	**0.965**
	SAM $\downarrow$	0.610	0.226	0.161	0.113	0.181	0.152	0.130	0.050	0.077	**0.043**	**0.041**

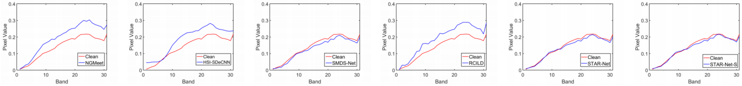
数值实验

(a) Clean/PSNR (b) Noisy/18.792 (c) BM3D/30.578 (d) BM4D/33.410 (e) LLRT/30.746 (f) LRTDTV/32.993

(g) NGMeet/31.510 (h) HSI-SDeCNN/32.141 (i) SMDS-Net/36.641 (j) RCILD/33.546 (k) STAR-Net/38.248 (l) STAR-Net-S/38.293



(a) Clean (b) Noisy (c) BM3D (d) BM4D (e) LLRT (f) LRTDTV



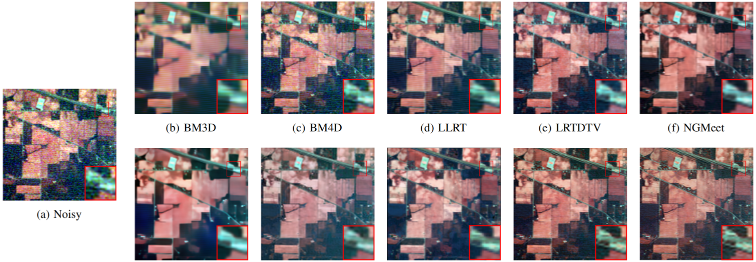
(g) NGMeet (h) HSI-SDeCNN (i) SMDS-Net (j) RCILD (k) STAR-Net (l) STAR-Net-S

WDC 数据集比较

σ	Index	Noisy	Model-based methods					Deep learning-based methods				
			BM3D	BM4D	LLRT	LRTDTV	NGMeet	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
10	PSNR ↑	28.561	36.142	38.435	39.540	40.887	42.712	40.756	43.616	28.968	**46.173**	**46.410**
	SSIM ↑	0.522	0.923	0.946	0.966	0.909	0.978	0.937	0.961	0.972	**0.980**	**0.982**
	SAM ↓	0.738	0.338	0.271	0.265	0.236	0.200	0.304	0.066	0.215	**0.043**	**0.042**
30	PSNR ↑	20.897	30.869	32.793	32.681	38.887	37.391	36.710	39.498	22.509	**39.924**	**39.950**
	SSIM ↑	0.150	0.784	0.834	0.858	0.888	0.854	0.832	0.916	0.882	**0.923**	**0.928**
	SAM ↓	1.020	0.519	0.445	0.387	0.335	0.260	0.399	**0.085**	0.243	**0.083**	**0.083**
50	PSNR ↑	17.778	28.882	30.901	30.470	35.250	36.160	35.062	36.014	22.160	**36.731**	**37.280**
	SSIM ↑	0.066	0.702	0.770	0.789	0.813	0.793	0.771	0.834	**0.872**	0.852	**0.871**
	SAM ↓	1.164	0.603	0.528	0.433	0.504	0.340	0.446	0.124	0.263	**0.122**	**0.121**
70	PSNR ↑	16.966	27.694	30.140	29.159	34.198	35.110	32.891	35.315	21.262	**35.964**	**36.286**
	SSIM ↑	0.051	0.657	0.740	0.761	0.776	0.761	0.602	0.811	**0.857**	0.831	**0.843**
	SAM ↓	1.205	0.652	0.560	0.433	0.547	0.500	0.980	0.135	0.290	**0.130**	**0.129**
Ave.	PSNR ↑	21.051	30.897	33.067	32.962	37.305	37.843	36.355	38.611	23.725	**39.698**	**39.982**
	SSIM ↑	0.197	0.766	0.823	0.844	0.847	0.846	0.786	0.881	0.896	**0.897**	**0.906**
	SAM ↓	1.032	0.528	0.451	0.380	0.406	0.325	0.532	0.102	0.253	**0.095**	**0.094**

数值实验

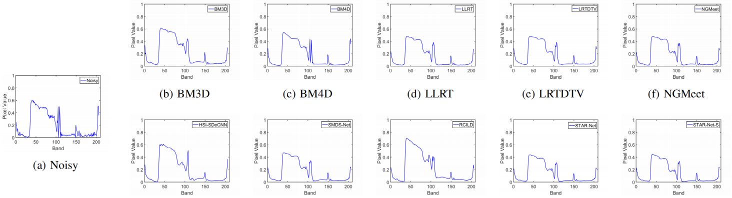
■ Indian Pines 数据集比较



(a) Noisy

(b) BM3D (c) BM4D (d) LLRT (e) LRTDTV (f) NGMeet

(g) HSI-SDeCNN (h) SMDS-Net (i) RCILD (j) STAR-Net (k) STAR-Net-S



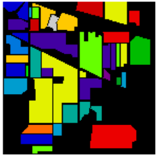
(a) Noisy

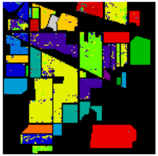
(b) BM3D (c) BM4D (d) LLRT (e) LRTDTV (f) NGMeet

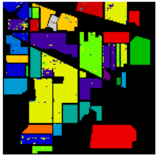
(g) HSI-SDeCNN (h) SMDS-Net (i) RCILD (j) STAR-Net (k) STAR-Net-S

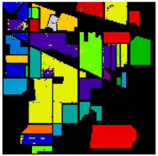
数值实验

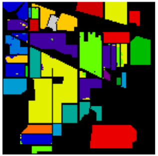
分类结果


(a) Ground truth

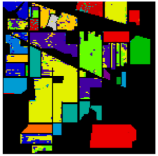

(b) Noisy

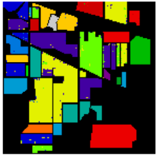

(c) BM3D

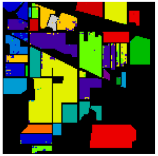

(d) BM4D

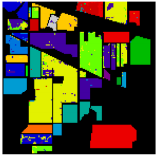

(e) LLRT

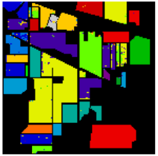

(f) LRTDTV

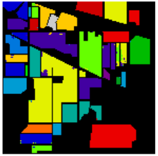

(g) NGMeet


(h) HSI-SDeCNN


(i) SMDS-Net


(j) RCILD


(k) STAR-Net


(l) STAR-Net-S

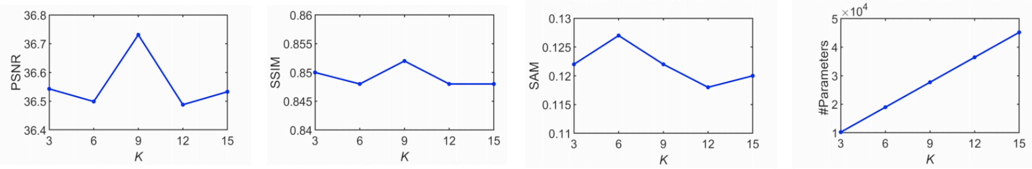
参数选择

Methods	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
#Parameters	1,892,100	**5,103**	2,892,288	**27,702**	28,487

时间比较

Methods	HSI-SDeCNN	SMDS-Net	RCILD	STAR-Net	STAR-Net-S
Time	**11.027**	293.606	**30.706**	233.106	238.366

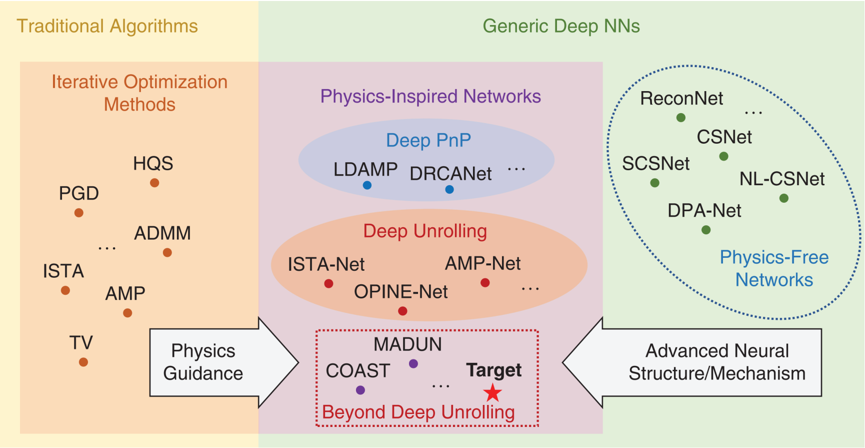
迭代次数比较



(a) PSNR (b) SSIM (c) SAM (d) #Parameters

小结

- https://deepinv.github.io/deepinv



Q&A

Thank you!

感谢您的聆听和反馈